

Understanding the Number System

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Part Two in the First
Module in the CARD
Algebra Series



Goals

This module identifies the real number system and concepts that move beyond arithmetic by introducing the use of negative numbers.

One should be able to

- recognize and compute absolute values
 - compute differences between number pairs on the number line
 - identify positive and negative values when computing all real numbers
 - perform complex operations
 - evaluate expressions containing letter values
- 

Real Numbers

- We learn to count **NATURAL NUMBERS**;
numbers greater than zero
- When adding, subtracting, multiplying or
dividing these numbers some answers result
in **WHOLE NUMBERS**
other answers result in **FRACTIONS**
- Mathematicians invented **NEGATIVE NUMBERS**
to represent when a large number is subtracted
from a smaller number and the answer falls
below zero
- Negative numbers, positive numbers and
zero are called **INTEGERS**

1, 2, 3,

0, 1, 2,

$\frac{1}{2}$, $\frac{3}{4}$,

.... -3, -2, -1

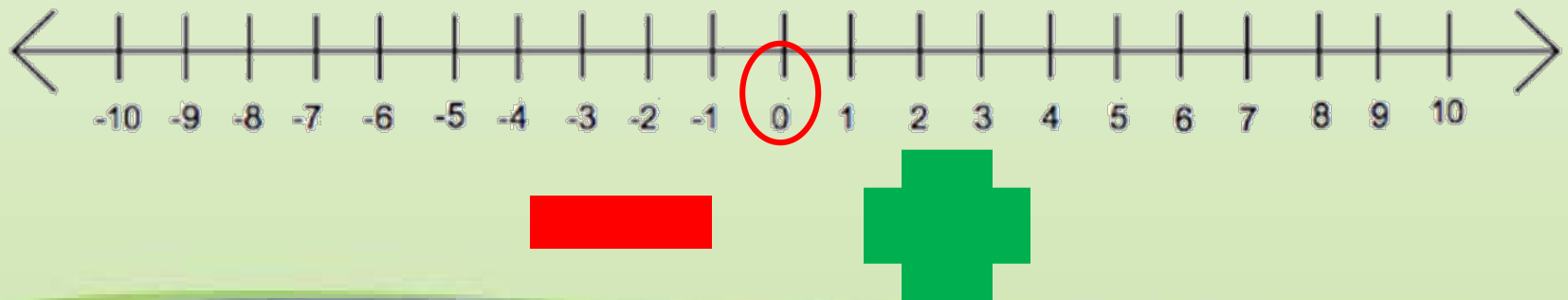




Number Line

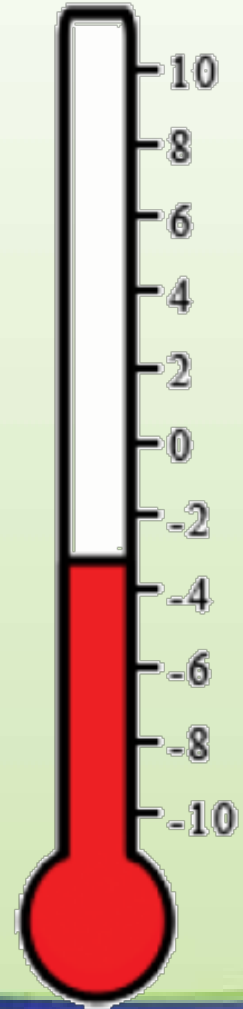
This is the representation of real numbers as points on a line.

This represents that every point corresponds to one and only one real number and every number corresponds to one and only one point.



Number Line aka Number Scale

- A thermometer is an example of a scale containing both positive and negative numbers.
- We refer to the numbers above zero as positive indicated by a plus sign (+) or no sign at all and the numbers below zero as negative as indicated with a minus sign (-).
- 10 and -10 above and below the zero are a set of OPPOSITES.



Real Numbers

- The entire collection of integers and positive and negative fractions are called **RATIONAL NUMBERS**
- The counterpart of rational numbers is **IRRATIONAL NUMBERS**, which are numbers that can not be expressed as a ratio of two numbers.

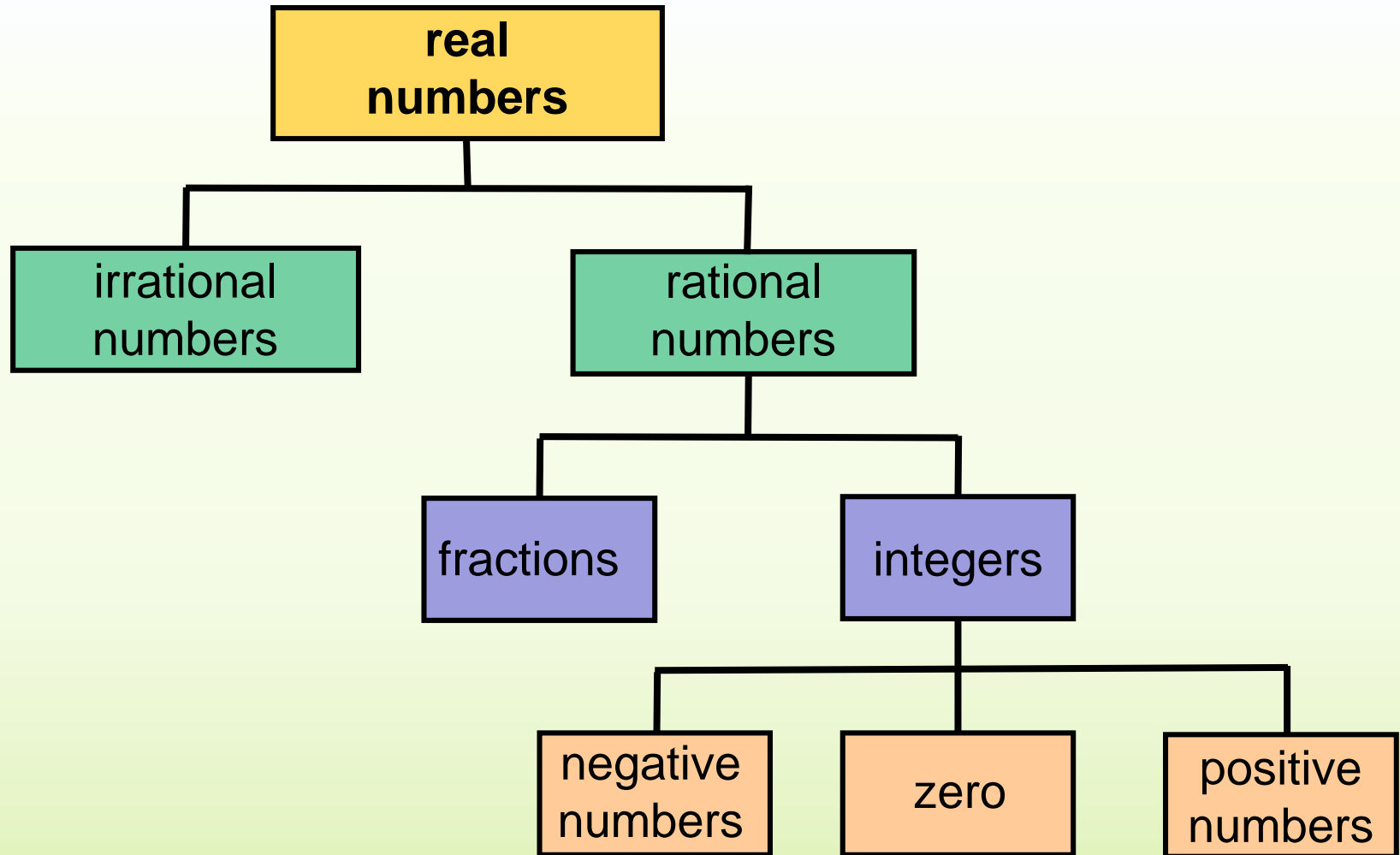
Two most common examples is pie or square root.

$$\pi \text{ (3.14159)} \quad \sqrt{9} = 3$$

- Rational and irrational numbers comprise **REAL NUMBERS**

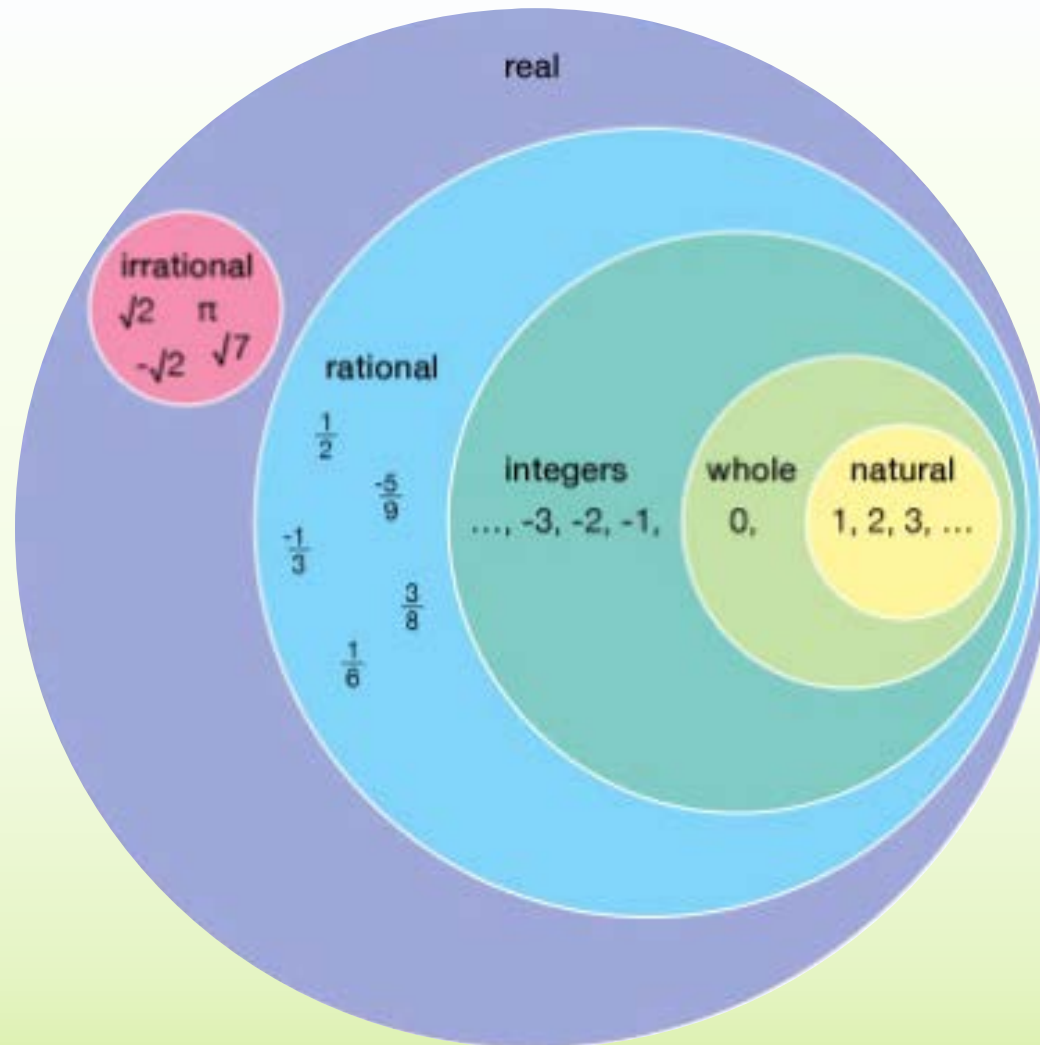
Real Number System

(represented in hierarchal graph)



Real Number System

(represented in a stacked Venn diagram to show overlap of relationships)

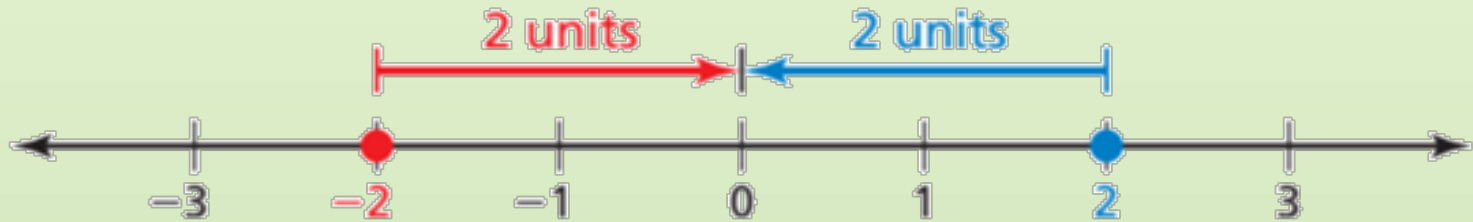


Absolute Value

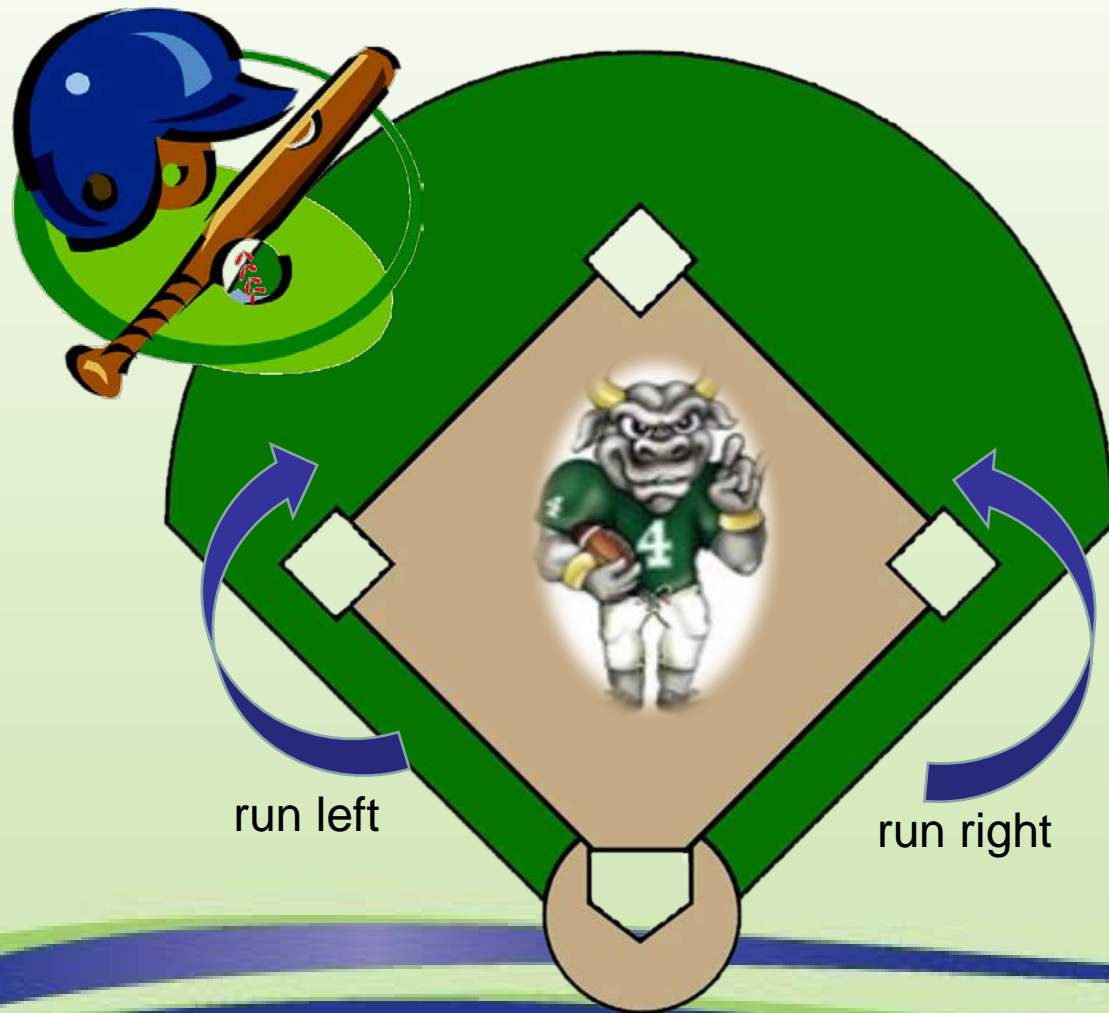
Words The **absolute value** of a number is the distance between the number and 0 on a number line. The absolute value of a number a is written as $|a|$.

Numbers

$$|-2| = 2 \quad |2| = 2$$



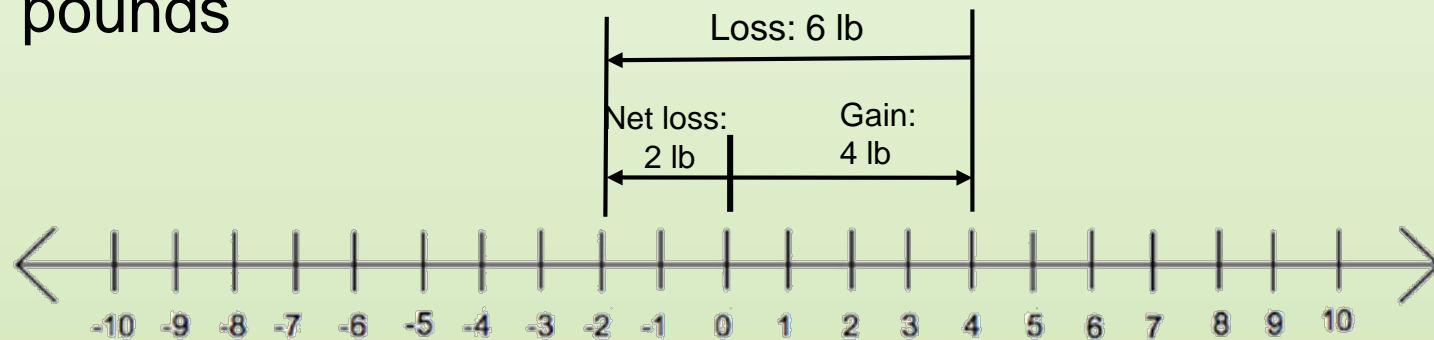
Absolute Value Comparison



Another way to look at an absolute value is when describing a baseball infield, the bases in little league are each 60 feet away from one another. So if you run clockwise (to the left) OR counter clockwise (to the right), it doesn't matter which direction you go the value is 60 ft between the bases.

Adding

- Adding in Algebra is also called COMBINING numbers
- Combining numbers is to obtain a single number to represent the total or combination of the two
- An example: someone gains four pounds then loses six pounds, the combined result is a loss of two pounds



Algebraic Solution $(+4) + (-6) = -2$



Rules: Addition

1. To add two numbers having like signs, add their absolute values and prefix their common sign.



$$-9 + (-7) \quad \text{or} \quad |9| + |7| = -16$$

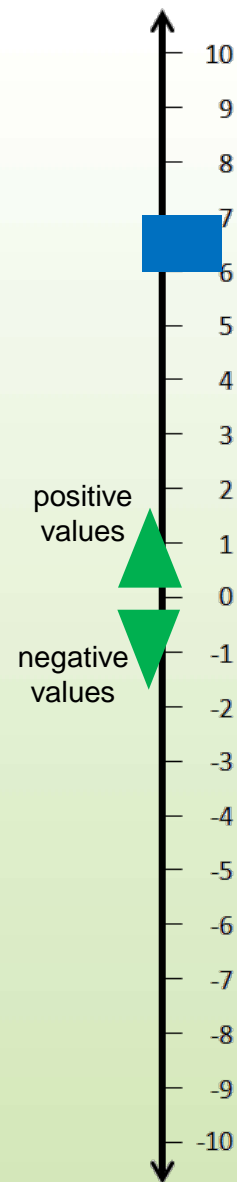
2. To add two numbers having opposite signs, find the difference of their absolute values and prefix the sign of the number having the larger absolute value.

$$9 + (-7) \quad \text{or} \quad |9| - |7| = +2$$

9 has a larger absolute value which is positive (+)

Subtracting

- Subtracting in Algebra is finding the *directed distance* from one position to another on a number scale.
- 2 concepts of *directed distance*:
 - **Interval** (distance) between two positions 
 - **Direction** of change (positive & negative) 



Subtraction Vocabulary

Minuend, the number another number is being subtracted from **5**

Subtrahend, the number being subtracted **- 3**
—

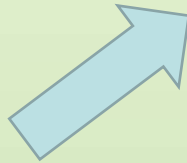
Difference, the answer in a subtraction problem **= 2**

Rules: Subtraction

3. To subtract a number, add its opposite (its additive inverse).



$$9 - (+7) = 9 + (-7) = 2$$





Rules: Multiplying

4. When multiplying two numbers, if the signs are the same, the product will be positive.

$$3 \times 4 = 12 \quad \text{or} \quad (-3) \times (-4) = 12$$

5. When multiplying two numbers, if the signs are different, the product will be negative.

$$(-3) \times (4) = -12 \quad \text{or} \quad (3) \times (-4) = -12$$

Rules: Multiplying (continued)

6. Regardless of the number of factors, the product of more than two numbers is always negative if there are an odd number of negative factors, and positive if there are an even number of negative factors.



Dividing

The method of determining the sign of the quotient in division is based on the rule of signs in multiplication (Rules 4,5 & 6).

$$- 4 \div (-2)$$

$$\frac{- 4}{- 2}$$

$$= 2$$

Rules: Evaluating Expressions



- 7 (a) To evaluate an expression, first substitute the values given for the letters, enclosing them in parenthesis.
- (b) Perform the indicated operations in the correct order.
1. Simplify inside the parentheses first;
 2. Next, apply any exponents;
 3. And finally, follow the procedural rules of adding positive and negative values.

$$2x - 5y = ? \quad \text{For } x = -3, y = +2$$

$$2(-3) - 5(+2) = ?$$

$$-6 - 10 = -16$$

References

- **Practical Algebra: A self teaching guide, 2nd edition** (1991). By Peter Selby & Steve Slavin. NY, NY John Wiley & Sons, Inc.
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